# Elegance Re-discovered and Preserved

by Frank J. Hatfield

The Charlotte Highway Bridge[1] (fig 1) probably did not require a strength determination before being re-deployed for pedestrian use. Fabricated by the Buckeye Bridge Company and erected in 1886 by H. P. Hepburn Engineering and Contracting, it likely conformed to a standard for strength[2] that exceeds the modern recommendation for pedestrian bridges[3]. Spanning 177 feet over the Grand River south of Portland, Michigan, it carried a single lane of traffic for more than a century, including trucks that flagrantly violated the posted weight limit. Finally replaced in 2001, the Charlotte Highway Bridge was dismantled and, after limited restoration, was re-assembled in the Calhoun County (Michigan) Historic Bridge Park[4]. Repair of a few damaged truss members, replication of the cross beams, and replacement of stringers and decking restored its original strength.

## Mathematical Mystery

More from curiosity than necessity, an analysis was performed to verify strength of the Charlotte Highway Bridge trusses (fig 2a), which are Whipple or double-intersection Pratt trusses characterized by diagonal ties that cross two panels but are not connected to the vertical strut between those panels. This was a routine task for a structural analysis computer program[5] but raised the question of how it was done in the late nineteenth century. Two methods[6] would have been available, one graphical and the other algebraic. Several difficulties were encountered when trying to apply these methods to the Whipple truss. First, the actual truss does not conform to the idealized definition of a truss because the end posts and top chord (L0-U1-U10-L11) are rigidly framed rather than being separate segments connected by pins at the intermediate joints (U1, U2, U3,…U10) which would allow relative rotation of the segments. This complication precludes the use of both methods but is resolved by assuming that the actual truss conforms to the idealized definition, an assumption that introduces minor inaccuracies, usually on the side of safety (i.e., over-estimation of forces acting on members). The second difficulty is that some of the diagonal ties may experience compression force. These members are designed to carry only tension force and therefore are so slender that when loaded in compression they bow or buckle, and, although not damaged, are ineffective. It is not always obvious which ties are in compression for any particular distribution of load on the deck. The third difficulty is that the truss has three more members than are required for stability. Ties that are loaded in compression can be considered to be missing but typically this eliminates only one or two of the redundant members. The presence of redundant members precludes both the graphical and algebraic methods. In short, there appeared to be no practical way to compute forces on members using nineteenth century design-office technology.

## Nineteenth Century Solution

However, the bridge designers of that time used an expedient approximation that represents the Whipple truss as two Pratt trusses[6] (figs 2 b and c), one of which includes the diagonal ties and vertical struts that terminate on odd-numbered joints, while the other includes the diagonal ties and vertical struts that terminate on even-numbered joints. End posts (U1-L0, U10-L11), upper chord (U1-U10), lower chord (L0-L11), and hip verticals (U1-L1, U10-L10) belong to both component trusses. Loads transferred to the truss at odd-numbered joints L3 through L9 are assigned to the odd component truss, loads transferred at even-numbered joints L2 through L8 are assigned to the even component truss, and loads transferred at L1 and L10 are divided equally between the two component trusses. Note that the odd component truss (fig 2b) is the same as the even component truss (fig 2c) flipped end-for-end so that an analysis of one is also an analysis of the other. The component Pratt truss has one more member than is required for stability but for any realistic load distribution it is apparent which one of the crossing ties in the central panel is in compression and therefore considered to be missing. Analysis by either of the nineteenth century methods becomes practical and gives member forces in diagonal ties and vertical struts directly, while forces in the ends posts, chords and hip verticals are determined by summing. For example, the force in member U2-U3 of the Whipple truss is the sum of forces in members U1-U3 and U7-U9 of the odd component truss (or U2-U4 and U8-U10 of the even component truss).

**Comparison to Computer Analysis**

The component Pratt truss can be analyzed by either nineteenth century method in less time than is needed to type all the input data for a computer analysis of the Whipple truss. However, the antique approach introduces two sources of inaccuracy. First, representing a Whipple truss as two superimposed component trusses is not quite valid because every rectangular panel of a Whipple truss is crossed by diagonal ties that belong to both the odd and even component trusses so that any force transmitted across a panel is shared by both component trusses, even if load is applied to only one. Second, the assumption of a segmented, pin-connected upper chord does not match the reality of rigidly framed upper chord and end posts. A computer analysis does not rely on these two approximations.

Results of nineteenth century and computer analyses were compared using a two-to-one ratio of live to dead load, corresponding approximately to a live load on the deck of 100 pounds per square foot and a dead load including all the iron members of the bridge plus typical timber decking. Since the Whipple truss is symmetric end-to-end, only the results for the left half are discussed. Geometric properties of members of the Charlotte Highway Bridge were required for the computer analysis but not for the older method. Comparison of computed maximum forces in the end posts, upper and lower chords and hip verticals revealed that the antique method under-estimated forces in some of those members by no more than 0.4% and over-estimated forces in the other members by no more than 1.0%. The older method was not as remarkably accurate for the vertical struts and diagonal ties. The forces in strut U3-L3 and ties U1-L3, U3-L5 and U4-L6 were the only ones to be under-estimated and then only by 0.4% to 3.3%. Inaccuracies not exceeding 5% are generally considered to be negligible compared to the uncertainty in estimates of future loads on a bridge. The inaccuracies in all the other struts and diagonal ties were on the side of safety and ranged from over-estimates of 3.8% up to nearly 30% for strut U5-L5 and tie U5-L7. Curiosity (again) motivated examination of those extreme discrepancies, both of which corresponded to live loads applied only at joints L7 through L10 and dead loads applied at all lower chord joints (L1 through L10). With this distribution of loads, ties U6-L4 and U7-L5 would carry compression forces if they were able. Since they are not, they were considered to be missing. The two sources of inaccuracy were disassociated by using two computer analyses, one with a rigidly framed upper chord and the other with a segmented, pin connected upper chord. These analyses showed that the nineteenth century practice of representing the Whipple truss as two superimposed Pratt trusses caused 66% of the discrepancy in the computed force in U5-L5 and 59% of the discrepancy in U5-L7. The assumption of a segmented, pin-connected upper chord accounted for the remaining smaller portions of the discrepancies. The two types of inaccuracy are additive for half the struts and ties, including U5-L5 and U5-L7, but offsetting for the others. Finally, it should be noted that even the largest discrepancies are of little consequence. For example, U5-L7 is a one-inch diameter bar; computer analysis justifies reducing its diameter by only 1/8 inch.

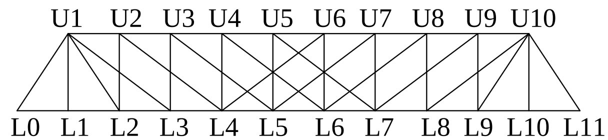
Clearly, the expedient nineteenth century practice of modeling a Whipple truss as two Pratt trusses produced a safe and sufficiently accurate design.

**Camber without Curvature**

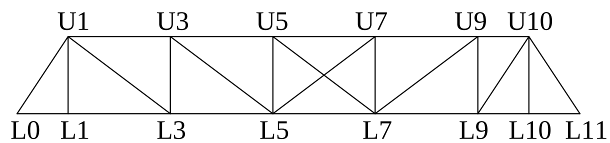
The iron workers who re-assembled the Charlotte Highway Bridge began by placing the cross beams on timber cribbing on a bluff above and adjacent to the park entry road that it would eventually span. The cross beams were approximately level with one another as the stringers and most of the truss members were added. There was no camber and the upper chord, consisting of five straight pieces, was essentially straight over its entire length. The upper chord pieces were connected by bolted splice plates at U3, U5, U6, and U8. The iron workers noted that the holes in these splice plates did not line up exactly with holes in the members. In order to fit the last member, an end post, into the truss, it was necessary to jack up some of the cross beams.

Originally, camber had been built into the truss by fabricating segments of the upper chord slightly longer than the nominal panel length, which was the usual practice[6]. For a Pratt truss, this causes the interior joints on the lower chord to define a smooth curve, assuming that the upper chord is segmented and pin-connected at every joint. With that assumption, a camber analysis for this Whipple truss predicted, surprisingly, that the joints on the lower chord would define five straight lines with angular declinations at L3, L5, L6 and L8. These points are directly below the splices for the upper chord, which explains why each of its five pieces could be straight. After the last member of the truss was fit into place by raising the cross beams, the bottom chord matched the predicted camber, there were similar angular declinations at the upper chord splices, and the bolt holes for those splices aligned perfectly.

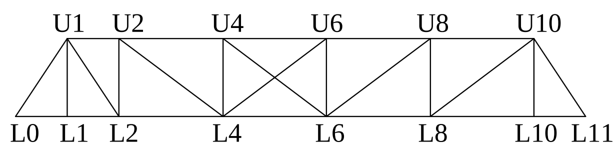
A 1920 textbook[6] states that “The Whipple truss was formerly quite generally used for long span highway and railway bridges, but is now rarely built...” Surviving examples are scarce but the restored Charlotte Highway Bridge will continue to display the elegant engineering of the Whipple truss for many more generations.




a. Whipple Truss




b. Odd Component Pratt Truss




c. Even Component Pratt Truss

**Figure 2.** Whipple Truss Represented as Two Pratt Trusses

**References**

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